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DATA ENVELOPMENT ANALYSIS- A TRIANGULAR PYTHAGOREAN FUZZY APPROACH

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ABSTRACT
Data Envelopment Analysis (DEA) is a technique for evaluating the relative efficiency or effectiveness of decision-making units (DMUs) with multiple inputs and outputs. In this article, an extended model of DEA is proposed under a triangular Pythagorean fuzzy environment, where the inputs and outputs of DMUs are represented by Triangular Pythagorean Fuzzy Numbers (TPFNs). A new strategy to solve the triangular Pythagorean DEA model is proposed. Finally, a numerical example is provided to illustrate the proposed method."

Key words: Data envelopment Analysis, efficiency, Decision Making Units, Triangular Pythagorean Fuzzy Number.

1 Introduction

1.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a well-known performance evaluation technique used to measure the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs. The traditional DEA, known as the CCR model, was proposed by Charnes, Cooper, and Rhodes in 1978. It calculates a ratio of output to input, referred to as the efficiency score. This ratio is determined by a weighted sum of the given outputs and inputs. The efficiency score of each DMU is computed by maximizing a linear programming problem, subject to the constraint that the ratio of output to input is less than or equal to one. The model was further extended by Banker, Charnes, and Cooper in 1984 to account for variable returns to scale. This extension, known as the BCC model, produces a convex curve that depicts a piecewise efficient frontier passing through all efficient DMUs.

The DEA model measures the relative efficiency of given DMUs based ongiven outputs (i.e., maximization problem) or given inputs (i.e.,minimizationproblem).DEA works by comparing the performance of each DMU to the best-performing DMUs in the sample. In other words, DEA evaluates the efficiency of each DMU relative to a hypothetical" best practice" unit that achieves the highest possible level of outputs for a given set of inputs. This makes DEA different from other efficiency analysis methods, such as regression analysis, which compare the performance of each DMU to a predetermined benchmark.

DEA can handle multiple inputs and outputs, and can be used to evaluateboth technical and scale efficiencies. Technical efficiency measures the ability of a DMU to produce a given level of outputs with the minimum amount of inputs, while scale efficiency measures the ability of a DMU to operate at the optimal scale of production. Fuzzy Data Envelopment Analysis (FDEA)is an extension of traditional DEA that incorporates uncertainty and imprecision in the input and output data. Fuzzy DEA is particularly useful in situations where the data is not fully known or the inputs and outputs are difficult to quantify precisely. In fuzzy DEA, the inputs and outputs of each decision-making unit (DMU) are represented by fuzzy numbers instead of crisp numbers. Fuzzy numbers are a type of number that allows for imprecision and ambiguity in the data. They are represented as a range of possible values with different degrees of membership, where each value represents a possible interpretation of the data. The efficiency scores in fuzzy DEA are calculated using a similar linear programming model as in traditional DEA. However, the weights assigned to the inputs and outputs are also represented as fuzzy numbers to account for uncertainty in the weights.

The fuzzy efficiency scores obtained from the optimization process can be interpreted as a range of possible efficiency levels, reflecting the imprecision and ambiguity in the data. Fuzzy DEA has several advantages over traditional DEA. It can handle imprecise and uncertain data, allowing fora more realistic representation of the data. It can also provide more robust and reliable results, as the fuzzy efficiency scores represent a range of possible outcomes rather than a single point estimate. However, fuzzy DEA also has some limitations, such as the need for expert knowledge

to specify the fuzzy numbers and the computational complexity of the optimization process.
Hatami-Marbini classified the FDEA models into four group approach and later expanded into six group approaches by Emroujnezad which are thetolerance approach, the α -cut approach, the fuzzy ranking approach, the fuzzy arithmetic approach, the possible approach, and type-2 fuzzy sets approach. Wang, Liu, and Liang constructed the efficiency modeling of FDEA with the help of fuzzy arithmetic operations. Kao and Liu formulated a pair of parametric FDEA models to derive the lower and upper bound on the membership function of efficiency score by using the cut approach. Based onthe α-cut approach, Sahil et al. derived an FDEA model to evaluate the efficiency score of DMUs in the presence of parabolic fuzzy inputs and outputs.

1.2 Method description

DEA is used to empirically measure productive efficiency of decision- making units (DMUs). It was based on research on effectiveness in terms of productivity by Farell in 1957, who used the scientific works of Debreu and Koopmans published in 1951 as the foundation of his own analyzes. This method is distinguished by its wide application in numerous sectors of the economy, including logistics, healthcare, as well as banking and economy. For example, for the assessment of the effectiveness of higher education, the DMU will be universities, and for insurance activities, the DMU will be insurance companies. The efficiency score of each DMU is calculated as the ratio of the weighted sum of outputs to the weighted sum of inputs.

$$Efficiency = \frac{Weighted\ sum\ of\ outputs}{Weighted\ sum\ of\ inputs}$$

The weights assigned to the inputs and outputs are chosen in such a way that the efficiency score of each DMU is maximized, subject to the constraint that the weights are non-negative. DMUs with an efficiency score of 1 are considered fully efficient, meaning that they use the minimum amount of inputs to produce the maximum amount of outputs, relative to the other DMUs in the sample. DMUs with an efficiency score less than 1 are considered inefficient, meaning that they could improve their performance by reducing their inputsor increasing their outputs, or both. DEA efficiency can be used to identify the best-performing DMUs and to determine the sources of inefficiency in the worst-performing DMUs.

2 Technique

Efficiency of a DMU is established as the ratio of sum weighted outputto sum weighted input, subjected to happen between one and zero.
Let a set of n DMUs, with each DMUj (j =1, 2,...n) by using m inputs
$x_{pj}(p = 1, 2, ...m)$ and producing s outputs y_{bj} (b =1,2,...s).
If DMUp is under consideration, the CCR model for the relative efficiency isthe following model:

$$\varphi_p{}^* = \max \frac{\sum_{b=1}^{s} u_b y_b}{\sum_{p=1}^{m} v_p x_p}$$

subject to the constraint

$$\frac{\sum_{b=1}^{s} u_b y_b}{\sum_{p=1}^{m} v_p x_p} \leq 1$$

$$u_b, v_p \geq 0 \ \forall \ b, p$$

Where $u_b(b = 1, 2, ...s)$ and $v_p(p = 1, 2, ...m)$ are the weights of the p^{th} input and b^{th} output. This fractional linear programming problem is calculated for each DMU to find out its best input and output weights.
To simplify the computation, the above nonlinear problem can be converted to a linear

programming (LP)and the model was called the CCR model:

$$\varphi_p{}^* = \max \sum_{b=1}^{s} u_b y_b$$

subject to the constraints

$$\sum_{p=1}^{m} v_p x_p = 1$$

$$\sum_{b=1}^{s} u_b y_b - \textstyle\sum_{p=1}^{m} v_p x_p \leq 0$$

$$u_b, v_p \geq 0 \ \forall\, b, p$$

Run this model n-times to work out the efficiency of n DMUs. The DMUp is efficient if $\varphi^* = 1$. Otherwise, it is inefficient.

3 Triangular Fuzzy Numbers

A triplet (m_1, m_2, m_3) is known as Triangular Fuzzy Number, where m_1 represents the smallest value, m_2 represent the most probable value and $m_3$represent the largest value of any fuzzy event.

3.1 Definition

Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number, then its membership function is defined as,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if\ x < a \\ \frac{x-a}{b-a} & if\ a \leq x < b \\ \frac{c-x}{c-b} & if\ b < x \leq c \\ 0 & if\ x > c \end{cases}$$

The mode b is the point where the membership value is maximum and represents the most probable value of the fuzzy number. The distance between a and c represents the spread or the degree of uncertainty of the fuzzy number. Triangular fuzzy numbers are commonly used in decision-making and optimization problems where uncertainty and imprecision exist. They can be used to model uncertain information, such as expert judgments or incomplete data.

3.2 Pythagorean fuzzy set

Pythagorean fuzzy set (PFS) specifically to handle the situations where the Intuitionistic Fuzzy Set (IFS) method falls short. PFS is an extension of IFS. The PFS extension improves both the flexibility and applicability of IFS.

3.2.1 Definition

Let X be a universal set. Then, a Pythagorean fuzzy set $\tilde{A}$ is defined by the following:

$\tilde{A} = \{(x, \mu_{\tilde{A}}, \nu_{\tilde{A}})/x \in X\}$ where the functions $\mu_{\tilde{A}}(x): X \to [0,1]$ and $\nu_{\tilde{A}}(x): X \to [0,1]$ define the degree of membership and the degree of non-membership respectively.

For every $x \in X$,

$$0 \leq (\mu_{\tilde{A}}(x))^2 + (\nu_{\tilde{A}}(x))^2 \leq 1$$

If $(\mu_{\tilde{A}}(x))^2 + (\nu_{\tilde{A}}(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to $\tilde{A}$ defined as

$$\pi_{\tilde{A}}(x) = \sqrt{1 - (\mu_{\tilde{A}}(x))^2 + (\upsilon_{\tilde{A}}(x))^2} \text{ and } \pi_{\tilde{A}}(x) \in [0,1].$$

It follows that $(\mu_{\tilde{A}}(x))^2 + (\upsilon_{\tilde{A}}(x))^2 + \left(\pi_{\tilde{A}}(x)\right)^2 = 1$

4 Triangular Pythagorean Fuzzy DataEnvelopment Analysis

In this section, DEA under triangular Pythagorean fuzzyenvironment is established.
Consider the inputs and outputs for the j[th] DMU as

$$\tilde{x}_{pj} = \left(\left({x_{pj}}^{e}, {x_{pj}}^{f}, {x_{pj}}^{g}\right), \mu_{pj}, \upsilon_{pj}\right)$$

$$\tilde{y}_{bj} = \left(\left({y_{bj}}^{e}, {y_{bj}}^{f}, {y_{bj}}^{g}\right), \mu_{bj}, \upsilon_{bj}\right)$$

which is the Triangular Pythagorean Fuzzy Numbers (TPFNs).
Then the triangular Pythagorean fuzzy CCR model that called TPFN-CCRis defined as follows:

$$\varphi_r{}^{*} = \max \sum_{b=1}^{s} u_b \widetilde{y_{br}}$$

subject to the constraints

$$\sum_{p=1}^{m} v_p \widetilde{x_{pr}} = 1$$

$$\sum_{b=1}^{s} u_b \widetilde{y_{bj}} - \sum_{p=1}^{m} v_p \widetilde{x_{pj}} \leq 0$$

$$u_b, v_p \geq 0 \ \forall\, b, p$$

4.1 A New Ranking Functions

4.1.1 Definition

One can compare any two TPFNs based on the ranking function.

Let $A = \left((a_1, a_2, a_3), \mu_A, \upsilon_A\right)$be a TPFN. Then,

$$R(A) = \frac{a_1 + a_2 + a_3}{6}\left({\mu_A}^2 + 1 - {\upsilon_A}^2\right)$$

4.1.2 Definition

Let $A_i = \left((a_1, a_2, a_3), \mu_A, \upsilon_A\right)$ be n TPFNs. Then the aggregation ranking function is as follows:

$$\tilde{R}\left(\sum_{i=1}^{n} A_i\right) = \left(1 + \min {\mu_{A_i}}^2 - \max {\nu_{A_i}}^2\right) \sum_{i=1}^{n} \frac{R(A_i)}{1 + {\mu_{A_i}}^2 - {\nu_{A_i}}^2}$$

$$\tilde{R}\left(\sum_{i=1}^{n} A_i\right) = \frac{\left(1 + \min {\mu_{A_i}}^2 - \max {\nu_{A_i}}^2\right)}{6} \sum_{i=1}^{n} a_{1_i} + a_{2_i} + a_{3_i}$$

Now to solve the TPFN-CCR model, proposing an algorithm for finding the optimal solution for each DMUs.

4.2 Algorithm

Step 1: Consider the DEA model,

$$\varphi_r{}^{*} = \max \sum_{b=1}^{s} u_b \widetilde{y_{br}}$$

s.t

$$\sum_{p=1}^{m} v_p \widetilde{x_{pr}} = 1$$

$$\sum_{b=1}^{s} u_b \widetilde{y_{bj}} - \sum_{p=1}^{m} v_p \widetilde{x_{pj}} \leq 0$$

$$u_b, v_p \geq 0 \ \forall \ b, p$$

the inputs and outputs of each DMU are TPFNs.
Step 2: The model of **Step 1** can be transformed into the following model,

$$\varphi_r{}^* = \max \sum_{b=1}^{s} u_b \left((y_{br}{}^e, y_{br}{}^f, y_{br}{}^g), \mu_{y_{br}}, \upsilon_{y_{br}} \right)$$

s.t

$$\sum_{p=1}^{m} v_p \left((x_{pr}{}^e, x_{pr}{}^f, x_{pr}{}^g), \mu_{x_{br}}, \upsilon_{x_{br}} \right) = \tilde{1}$$

$$\sum_{b=1}^{s} u_b \left((y_{br}{}^e, y_{br}{}^f, y_{br}{}^g), \mu_{y_{br}}, \upsilon_{y_{br}} \right) - \sum_{p=1}^{m} v_p \left((x_{pr}{}^e, x_{pr}{}^f, x_{pr}{}^g), \mu_{x_{br}}, \upsilon_{x_{br}} \right) \leq 0$$

$$u_b, v_p \geq 0 \ \forall \ b, p$$

Step 3: Transform the above model into the following model,

$$\widetilde{R} \ (\varphi_r{}^*) = \max \widetilde{R} \sum_{b=1}^{s} u_b \left((y_{br}{}^e, y_{br}{}^f, y_{br}{}^g), \mu_{y_{br}}, \upsilon_{y_{br}} \right)$$

s.t

$$\widetilde{R} \left(\sum_{p=1}^{m} v_p \left((x_{pr}{}^e, x_{pr}{}^f, x_{pr}{}^g), \mu_{x_{br}}, \upsilon_{x_{br}} \right) \right) = \widetilde{R} \left(\tilde{1} \right)$$

$$\widetilde{R} \left(\sum_{b=1}^{s} u_b \left((y_{br}{}^e, y_{br}{}^f, y_{br}{}^g), \mu_{y_{br}}, \upsilon_{y_{br}} \right) \right) \leq \widetilde{R} \left(\sum_{p=1}^{m} v_p \left((x_{pr}{}^e, x_{pr}{}^f, x_{pr}{}^g), \mu_{x_{br}}, \upsilon_{x_{br}} \right) \right)$$

$$u_b, v_p \geq 0 \ \forall \ b, p$$

Step 4: Convert the above model into the following crisp model,

$$\varphi_r{}^* = \max \left[\sum_{b=1}^{s} \frac{\left(1 + \min\limits_{1 \leq b \leq s} \mu_{y_{br}}{}^2 - \max\limits_{1 \leq b \leq s} \nu_{y_{br}}{}^2 \right)}{6} \right] \sum_{b=1}^{s} u_b (y_{br}{}^e + y_{br}{}^f + y_{br}{}^g)$$

s.t

$$\frac{\left(1 + \min\limits_{1 \leq p \leq m} \mu_{x_{pj}}{}^2 - \max\limits_{1 \leq p \leq m} \nu_{x_{pj}}{}^2 \right)}{6} \sum_{p=1}^{m} v_p \left(x_{pr}{}^e + x_{pr}{}^f + x_{pr}{}^g \right) = 1$$

$$\frac{\left(1 + \min\limits_{1 \leq b \leq s} \mu_{y_{br}}{}^2 - \max\limits_{1 \leq b \leq s} \nu_{y_{br}}{}^2 \right)}{6} \sum_{b=1}^{s} u_b (y_{br}{}^e + y_{br}{}^f + y_{br}{}^g)$$

$$\leq \frac{\left(1 + \min\limits_{1 \leq p \leq m} \mu_{x_{pj}}{}^2 - \max\limits_{1 \leq p \leq m} \nu_{x_{pj}}{}^2 \right)}{6} \sum_{p=1}^{m} v_p \left(x_{pr}{}^e + x_{pr}{}^f + x_{pr}{}^g \right)$$

Step 5: Run the crisp model of **Step 4** and obtain the optimal solution.

4.3 Numerical Experiment

For the purpose of interpreting the practicability and the feasibility ofthe new method proposed in this research article, a numerical example isemployed.

There are five DMUs that consume two inputs to produce two outputs. Theseinputs and outputs are given by triangular Pythagorean fuzzy numbers.

The below table provides the data for this example.

DMU	**DMU1**	**DMU2**	**DMU3**	**DMU4**	**DMU5**
Input 1	⟨ (3.5,4,4.5); 0.7,0.3⟩	⟨(2.9,2.9,2.9); 0.6,0.2⟩	⟨(4.4,4.9,5.4); 0.6,0.1⟩	⟨(3.4,4.1,4.8); 0.4,0.2⟩	⟨(5.9,6.5,7.1); 0.7,0.3⟩
Input 2	⟨(1.9,2.1,2.3); 0.4,0.5⟩	⟨(1.4,1.5,1.6); 0.8,0.1⟩	⟨(2.2,2.6,3.0); 0.7,0.2⟩	⟨(2.2,2.3,2.4); 0.7,0.2⟩	⟨(3.6,4.1,4.6); 0.9,0.1⟩
Output1	⟨(2.4,2.6,2.8); 0.9,0.1⟩	⟨(2.2,2.2,2.2); 0.9,0.0⟩	⟨(2.7,3.2,3.7); 0.7,0.2⟩	⟨(2.5,2.9,3.3); 0.7,0.1⟩	⟨(4.4,5.1,5.8); 0.8,0.2⟩
Output2	⟨(3.8,4.1,4.4); 0.8,0.1⟩	⟨(3.3,3.5,3.7); 1.0,0.0⟩	⟨(4.3,5.1,5.9); 0.7,0.1⟩	⟨(5.5,5.7,5.9); 0.4,0.1⟩	⟨(6.5,7.4,8.3); 0.5,0.2⟩

Now, use the proposed algorithm to solve the performance assessment problem.
Construct a DEA model with mentioned TPFNs and using the proposedalgorithm to obtain the optimal solution.
First taking the case of DMU1.

$\varphi_1{}^* = \max\langle 2.4, 2.6, 2.8;\ 0.9, 0.1\rangle\, u_1 \ +\ \langle 3.8, 4.1, 4.4;\ 0.8, 0.1\rangle u_2$

s.t

$\langle 3.5, 4, 4.5; 0.7, 0.3\rangle v_1 + \langle 1.9, 2.1, 2.3; 0.4, 0.5\rangle v_2 = \tilde{1}$

$\langle 2.4, 2.6, 2.8; 0.9, 0.1\rangle u_1 + \langle 3.8, 4.1, 4.4; 0.8, 0.1\rangle u_2$
$\leq \langle 3.5, 4.0, 4.5; 0.7, 0.3\rangle v_1 + \langle 1.9, 2.1, 2.3; 0.4, 0.5\rangle v_2$

$\langle 2.2, 2.2, 2.2; 0.9, 0.0\rangle u_1 + \langle 3.3, 3.5, 3.7; 1.0, 0.0\rangle u_2$
$\leq \langle 2.9, 2.9, 2.9; 0.6, 0.2\rangle v_1 + \langle 1.4, 1.5, 1.6; 0.8, 0.1\rangle v_2$

$\langle 2.7, 3.2, 3.7; 0.7, 0.2\rangle u_1 + \langle 4.3, 5.1, 5.9; 0.7, 0.1\rangle u_2$
$\leq \langle 4.4, 4.9, 5.4; 0.6, 0.1\rangle v_1 + \langle 2.2, 2.6, 3.0; 0.7, 0.2\rangle v_2$

$\langle 2.5, 2.9, 3.3; 0.7, 0.1\rangle u_1 + \langle 5.5, 5.7, 5.9; 0.4, 0.1\rangle u_2$
$\leq \langle 3.4, 4.1, 4.8; 0.4, 0.2\rangle v_1 + \langle 2.2, 2.3, 2.4; 1.0, 0.0\rangle v_2$

$\langle 4.4, 5.1, 5.8; 0.8, 0.2\rangle u_1 + \langle 6.5, 7.4, 8.3; 0.5, 0.2\rangle u_2$
$\leq \langle 5.9, 6.5, 7.1; 0.7, 0.3\rangle v_1 + \langle 3.6, 4.1, 4.6; 0.9, 0.1\rangle v_2$

$u_b, v_p \geq 0$ b, i = 1, 2

Based on **Step 4** of algorithm, Convert the above model to the followingmodel

$$\varphi_1{}^* = \max\left[\frac{(1+0.64-0.01)}{6}\langle 2.4+2.6+\ 2.8\rangle u_1 \ +\ \langle 3.8+4.1+4.4\rangle u_2\ \right]$$

$$= \max[0.2717\langle 7.8u_1 + 12.3u_2\ \rangle]$$

$$\varphi_1{}^* = \max[2.1192u_1 + 3.3419u_2]$$

s.t

$$\frac{(1+0.16-0.25)}{6}\langle 3.5+4.0+\ 4.5\rangle v_1 \ +\ \langle 1.9+2.1+2.3\rangle v_2\ = 1$$

$.1517(12v_1 + 6.3v_2) = 1$

$1.8204v_1 + 0.95571v_2 = 1$

$$\frac{(1+0.64-0.01)}{6}\langle 2.4, 2.6, 2.8\rangle u_1 \ +\ \langle 3.8, 4.1, 4.4\rangle u_2 \leq \frac{(1+0.16-0.25)}{6}\langle 3.5, 4.0, 4.5\rangle v_1 \ +\ \langle 1.9, 2.1, 2.3\rangle v_2$$

$1.63(7.8u_1 + 12.3u_2) \leq 0.91(12v_1 + 6.3v_2)$
$12.714u_1 + 20.049u_2 - 10.92v_1 - 5.733v_2 \leq 0$
Similarly, for other constraints
$11.946u_1 + 19.005u_2 - 11.484v_1 - 5.94v_2 \leq 0$
$13.92u_1 + 22.185u_2 - 19.404v_1 - 10.296v_2 \leq 0$
$10.005u_1 + 19.665u_2 - 13.776v_1 - 7.728v_2 \leq 0$
$18.513u_1 + 26.862u_2 - 27.3v_1 - 17.22v_2 \leq 0$
Finally getting a model for DMU1 as;

$$\varphi_1^* = \max\,[2.11926u1 + 3.34191u2]$$

s.t
$1.8204v_1 + 0.95571v_2 = 1$
$12.714u_1 + 20.049u_2 - 10.92v_1 - 5.733v_2 \leq 0$
$11.946u_1 + 19.005u_2 - 11.484v_1 - 5.94v_2 \leq 0$
$13.92u_1 + 22.185u_2 - 19.404v_1 - 10.296v_2 \leq 0$
$10.005u_1 + 19.665u_2 - 13.776v_1 - 7.728v_2 \leq 0$
$18.513u_1 + 26.862u_2 - 27.3v_1 - 17.22v_2 \leq 0$
After computation with LiberOffice Calc, obtain the optimal solution for DMU1is 1.000.
i.e.,
$\varphi_1^* = 1.00$
Similarly finding the optimal solution for all DMUs.
Ranking of the five DMUs based on the efficiency values.

DMUs	**DMU1**	**DMU2**	**DMU3**	**DMU4**	**DMU5**
Efficiency	**1.00**	**0.91**	**0.62**	**0.77**	**0.58**
Ranking	**1**	**2**	**4**	**3**	**5**

5 Conclusion

In this research article, an extended model of DEA is proposed to handle performance evaluation problems under the TPFNs environment. The existing arithmetic operations are cannot take into account the interaction betweennon-membership function and membership function of different TPFNs, a new ranking function is proposed in this article to address the existing problem. A novel algorithm is developed to use these ranking functions to calculate the weight of each evaluation value in DMUs, which can effectively avoidunreasonable evaluation values.
Finally, use an example to illustrate the practicability and validity of the proposed method. In comparison with the classical and fuzzy DEA method, the significant characteristic of the extended DEA method is that it can handle the triangular Pythagorean fuzzy information simply and effectively. Among the five DMUs, DMU1 is more efficient than others.

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